OPTIMAL VORTEX FORMATION IN BIOLOGICAL PROPULSION

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Abstract

The formation of Vortex Rings generated through impulsively started jets studied experimentally. Vortex Rings which facilitate mass transfer by stationary pumps (e.g. cardiac chambers) and momentum transfer by mobile systems (e.g. jet-propelled swimmers). Previous research has shown that vortex rings generated in the laboratory can be optimized for efficiency or thrust, based on the jet length-to-diameter ratio (L/D) known as "formation number" lie in the range of 3.6 to 4.5 for a broad range of flow conditions. Additionally, optimal vortex formation can provide a framework in which to design engineered propulsions systems that are constrained by pressures unrelated to biology. Finally, analyze the relationship between optimal vortex formation and previously observed constraints on Strouhal frequency during animal locomotion in air and water. It is proposed that the Strouhal frequency constraint is but one consequence of the process of optimal vortex formation and that others remain to be discovered.

Key Words: Vortex rings, swimming, flying, cardiovascular flows, hydrodynamics

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1.Introduction

Jet flows in biological systems are typically created by the action of positive displacement pumps, which eject fluid from a source chamber through a nozzle or orifice (Vogel 1994). The flow regime of the jet can be characterized by the velocity of fluid exiting the chamber (U), the diameter of the jet at the chamber exit (D) and the kinematic viscosity of the fluid (v). For Reynolds numbers Re=UD/ v>6 of rapid jet acceleration during flow initiation causes the leading portion of the jet to roll into a toroidal fluid mass known as a vortex ring (Cantwell 1986; see figure 1). Laboratory experiments have demonstrated that the leading vortex makes a proportionally larger contribution to mass and momentum transport than an equivalent straight jet of fluid (Krueger & Gharib 2003; Dabiri & Gharib 2004). This fact, along with the discovery that physical processes terminate growth of the leading vortex ring at a jet length-to-diameter ratio (L/D) between 3.5 and 4.5 (Gharib et al. 1998; Mohseni & Gharib 1998), has spurred interest in the possibility that biological systems may optimize vortex formation for effective fluid transport (Gharib et al. 1998; Linden & Turner 2001; Mohseni, Ran and Colonius 2001; Krueger & Gharib 2003; Dabiri & Gharib 2004; Linden & Turner 2004). Inferences drawn from vortex rings generated in the laboratory are limited due to kinematic constraints on the vortex generators. Typically the vortices are created by ejecting fluid through a tube with constant exit diameter (Didden 1979; Gharib et al. 1998; Krueger & Gharib 2003; Dabiri & Gharib 2004). This is in contrast with the complex, time-dependant kinematics of positive displacement pumps found in nature (Vogel 1994). In situ measurements of fluid jet L/D (hereafter referred to as the dimensionless 'formation time' following Gharib et al. 1998) have suffered from an inability to unambiguously incorporate the observed time-varying exit diameter D(t). Attempts to reduce the observed kinematics using a time-averaged exit diameter ($\overline{D}$) have been inconclusive in determining a clear correlation between vortex formation dynamics and animal kinematics (Linden & Turner 2004; Anderson & Grosenbaugh 2005). The average jet diameter is by itself an insufficient index of jet kinematics because it lacks critical information regarding temporal trends in the jet exit diameter. As mentioned above, laboratory experiments have pointed to the existence of a critical formation time, after which growth of the leading vortex ring ceases and any additional fluid ejected takes the formof a trailing straight jet (Gharib et al. 1998). Since this vortex-limiting formation time is dictated by the time-history of the jet exit diameter and not its average value, an effective kinematic index must preserve this information. From this we can

properly record how jet flow is manipulated by changes in the exit diameter both before and after the vortex-limiting formation time is reached.

The goal of this paper is to combine laboratory experiments, in situ observations and a framework that reduces the kinematics to a single parameter in order to show that individual animal kinematics can be tuned in correlation with optimal vortex ring formation.

2. Methods

(a)Kinematic Analysis:

A suitable kinematic parameter to describe observed animal motions can be derived by considering an infinitesimal increment in the formation time $\widehat{T}$ (L/D) . It is defined based on the instantaneous vortex strength Γ(i.e., circulation, m2 s−1), as well as the strength ΔU (i.e ,U − Uambient, m s−1) and characteristic length scale D (m) of the shear layer feeding the vortex:

$$\widehat{T} = \frac{C\,\Gamma}{D\Delta U} \tag{1}$$

The constant factor C depends on the physical configuration of the vortex generator. More specifically, C is given by the inverse of the dimensionless vorticity flux $d\widehat{\Gamma}$ /$d\widehat{T}$ from a given vortex generator configuration.

We can approximate the circulation Γ generated by piston – cylinder apparatus using the slug model (Didden 1979, Lim & Nickels 1995), which assumes that within the boundary layer of thickness $\delta \ll D$, the wall-normal velocity component v is much smaller than the streamwise component u. Also, the flow outside the boundary layer is assumed to be spatially uniform and equal to a constant U, the jet velocity. Under these conditions , we may write the circulation as

$$\Gamma \approx h\int_0^T \int_{D/2-\delta}^{D/2} u w_\emptyset \, dydt \approx \int_0^T \int_{D/2-\delta}^{D/2} u\left(-\frac{du}{dy}\right) dydt = \frac{1}{2} U^2 T \tag{2}$$

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where ωφ is the azimuthal component of vorticity, y is the wall-normal direction (–D/2 < y < D/2

across the cylinder exit plane), and T is the duration of the vorticity flux from the piston-cylinder apparatus.

We can combine the result from Equation 2 with the known piston-cylinder shear layer strength

(ΔU =U − Uambient = U) and length scale (taken as the cylinder exit diameter D) to arrive at a

dimensionless vortex formation timescale based on the physical parameters of the vortex generator:

$$\widehat{T} = \frac{2\left(\frac{1}{2}U^2T\right)}{D\,U} = \frac{UT}{D}$$

For consistency in the following discussion, we use t to denote the advancing dimensional time

during vortex formation and T to denote the final dimensional time at the end.

(b)Laboratory apparatus:

To test this result experimentally, we studied jet flow vortex formation by creating an apparatus that incorporates an exit nozzle with a controllable, variable diameter on the flow tube of a traditional laboratory vortex generator (figure 1; cf. Dabiri & Gharib in press). This technique was used to probe the effects of a temporally increasing exit diameter on jet flows as is observed, for example, during tail-first swimming motions of squid (O'Dor 1988; Anderson & DeMont 2000; Bartol et al. 2001; Anderson & Grosenbaugh 2005) and the early refilling phase of ventricular diastole (Verdonck et al. 1996). Fluid forces were determined from the time-integrated jet thrust, or fluid impulse, $I = \frac{1}{2}\,\rho\,\int_V x \times (\nabla \times u)dv.$ This first moment of vorticity was computed given the position vector (x) of fluid particles relative to the nozzle exit plane and

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axis of symmetry, the velocity field (u) measured using digital particle image velocimetry (Willert & Gharib 1991) and the measurement volume (V).

3. Results

(a)Laboratory results:

The study of vortex ring formation has a rich history, much of which is well described in reviews

by Shariff & Leonard (1992) and Lim & Nickels (1995); therefore I do not revisit it in detail here.

However, it is noteworthy that, up to and including these reviews, this process was almost entirely studied for short dimensionless vortex formation times $\widehat{T} < 4$ (e.g., Auerbach 1991; Baird et al. 1977; Bernal & Kwon 1989; Didden 1979; Glezer & Coles 1990; Maxworthy 1972, 1977; Sallet & Widmayer 1974; Schatzle 1987;Weigand&Gharib 1994). Gharib et al. (1998) examined the vortex formation process for longer times, $\widehat{T} > 4$, and observed a robust limit on the maximum growth of vortex rings formed using a piston-cylinder apparatus. As illustrated in Figure 1, the forming vortex ring accepts the vorticity flux from the piston-cylinder apparatus until a dimensionless formation time of $\widehat{T} \approx 4$ is reached. Beyond this time, additional vorticity flux is rejected by the vortex ring and instead forms secondary vortices akin to a Kelvin-Helmholtz instability. Gharib et al. (1998) termed this process vortex ring pinch-off. This process is not to be confused with the physical
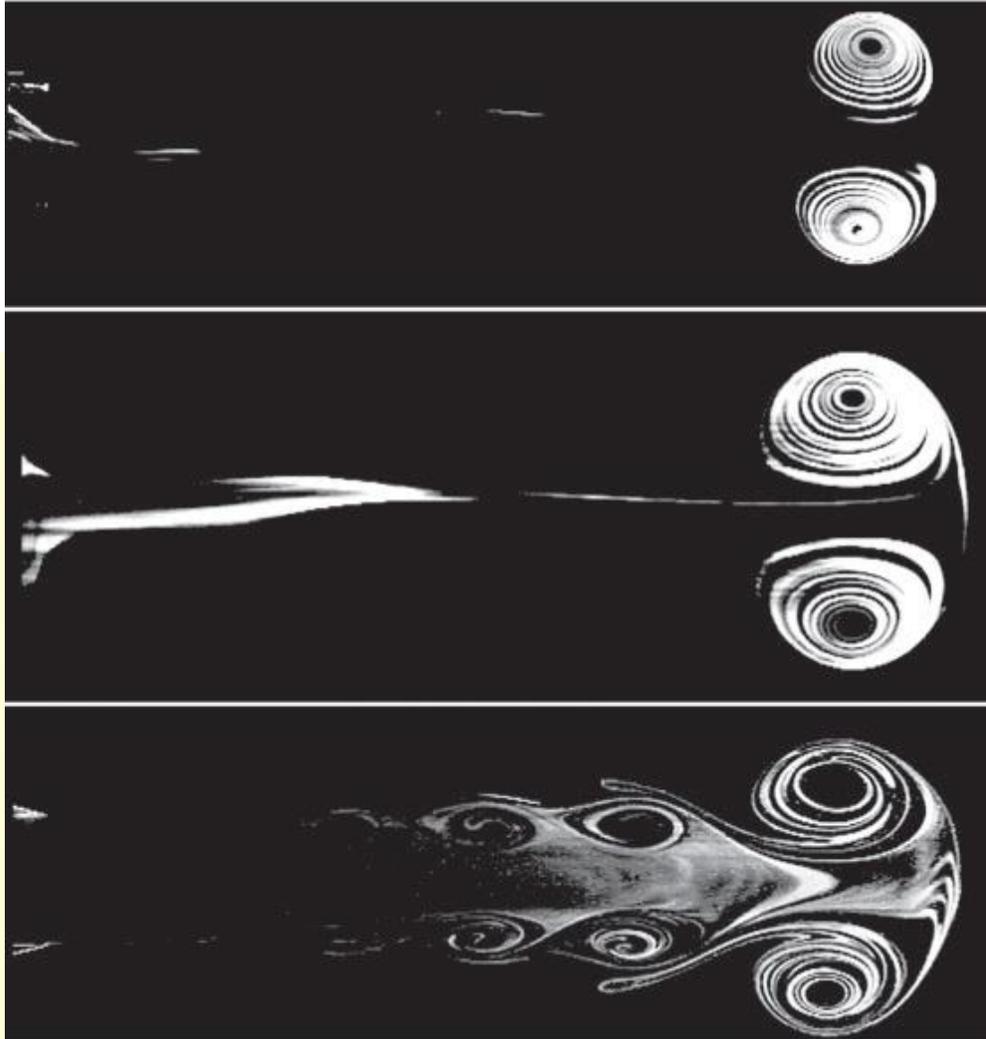
Figure 1. Visualization of vortex rings at downstream position X/D ≈ 9 for
(a) $\widehat{T}$ = 2, (b) $\widehat{T}$ = 3.8, and (c) $\widehat{T}$ = 14.5. Figure taken from Gharib et al. 1998.

separation between the vortex ring and the trailing shear layer, which may occur later or not at all.

(b) Biological optimization strategy:

Given that vortex ring growth is limited by the Kelvin-Benjamin variational principle, one may

ask whether there are beneficial properties of vortex ring formation that scale with increasing vortex size. If so, then to achieve those benefits maximally, we wish to maximize the size of each generated vortex ring given the aforementioned constraint imposed by the dimensionless vortex

formation time $\widehat{T}$. This is the essence of optimal vortex formation: The quantity that is optimized per se is that which scales with increasing vortex ring size.

In a seminal work, Krueger (2001) experimentally deduced some of the properties of vortex ring formation that scale with increasing vortex size and are therefore potential targets for optimal vortex formation. He determined that the normalized time-averaged thrust per jet pulse, $\widehat{F}= \overline{F}/\rho AU2$ (where ρ is the fluid density, A is the cylinder exit area, and the overbar denotes a time average), was one such parameter, reaching its maximum value just before the limiting dimensionless vortex formation time $\widehat{T} \approx 4$ (Figure 2).
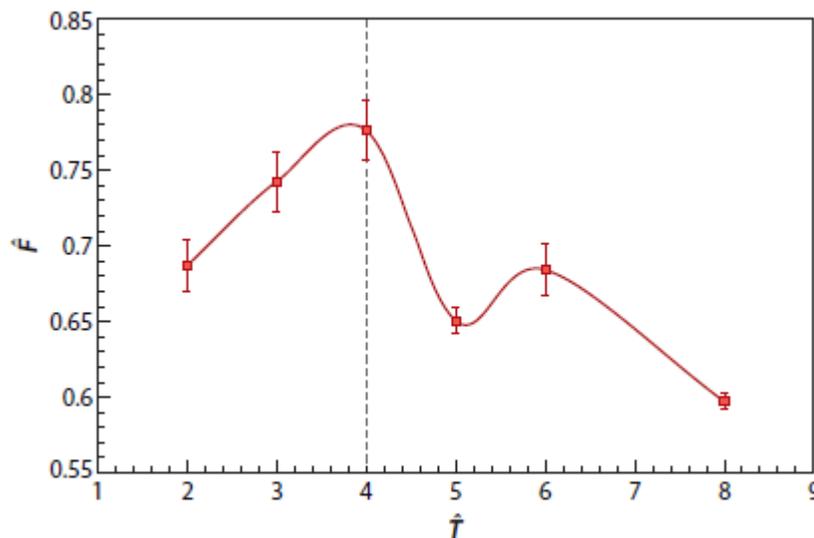

Figure 2. Normalized average thrust per pulse measurements. Figure taken from Krueger & Gharib 2003.

For a constant cylinder exit velocity U, the denominator of $\widehat{F}$ can be multiplied by U, resulting in a parameter that is effectively a measure of the energetic efficiency of propulsion via vortex formation. In other words, by maximizing the size of the vortex rings formed during propulsion, the efficiency of momentum transport is also optimized. Physically, the benefit of vortex ring formation for propulsion arises because of the entrainment of ambient fluid by the forming vortex ring (Auerbach 1991, Dabiri & Gharib 2004b, Olcay & Krueger 2007, Shadden et al. 2007), as well as the added mass of nonentrained fluid surrounding the vortex that must be

accelerated with the vortex ring (Baird et al. 1977, Krueger & Gharib 2003,Weihs 1977). This latter effect is mathematically equivalent to the added mass carried with a solid body in a potential flow, and it can be computed in terms of the velocity potential φ of the flow outside the vortex (Dabiri 2006). The propulsive reaction force experienced by the vortex generator is proportional to the sum of the shear layer source fluid, entrained fluid, and the added mass of the fluid surrounding the vortex ring (Dabiri 2005, Krueger 2001).

(c) Comparison with in vivo cardiac measurements:

The left heart is responsible for the collection of oxygenated blood from the pulmonary circulation and the subsequent distribution of that blood throughout the body. A key process within the left heart is the transfer of blood from the left atrium to the left ventricle via a starting jet that forms during early diastole (relaxation) of the left heart. During initiation of this starting jet, a vortex forms downstream from the mitral valve that separates the left atrium and left ventricle (Domenichini et al. 2005, Kim et al. 1995). This process of blood transport correlates with the overall health of the heart (e.g., Kilner et al. 2000); however, existing metrics that quantify cardiac health do not specifically account for the vortex formation process. Gharib et al. (2006) used the optimal vortex formation concept to quantify vortex formation in the left heart of patients measured using echocardiography. In a blind study
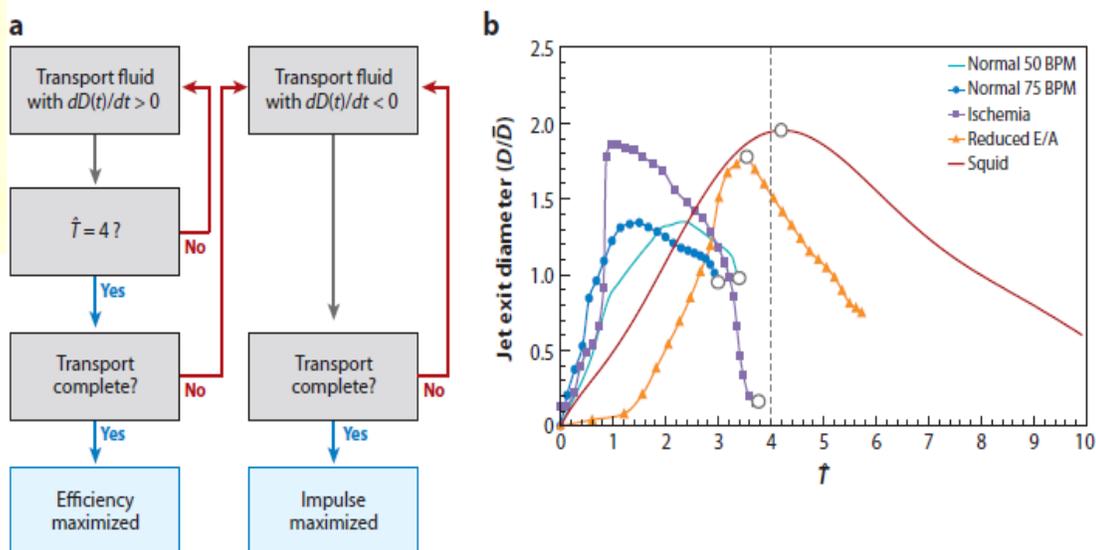
Figure 3.

(a) Optimal vortex formation protocols for maximizing efficiency or impulse.

(b) Measurements of mitral valve exit diameter D versus $\hat{T}$ for various left ventricle conditions and measurements of funnel diameter D versus $\hat{T}$ for squid jet propulsion. Initial funnel diameter is subtracted from the measurements for normalization. Open circles indicate points of correlation with optimal vortex formation time based on panel a. E/A is the ratio of blood-flow velocities during early diastolic filling and atrial contraction, respectively. Panel b adapted from Dabiri & Gharib 2005b. of 120 normal volunteers, they observed that vortex formation in the left ventricle occurred in a dimensionless time $\hat{T} \approx 4$, similar to the result of previous laboratory studies of vortex ring formation. Furthermore, patients with abnormal left heart function (i.e., dilated cardiomyopathy) exhibited significantly lower vortex formation times, indicating suboptimal fluid transport.

Gharib et al. (2006) used time-averaged values of the mitral valve exit diameter owing to the limited temporal resolution of the echocardiogram measurements. Dabiri & Gharib (2005b) hypothesized that the temporal variations in the characteristic length scale of the shear layer D(t) could provide additional information regarding the efficacy of fluid transport. They proposed a protocol based on the limiting dimensionless vortex formation time $\hat{T}$ and the time-varying length scale D(t) that could maximize either efficiency or impulse (Figure 3a). There are two basic tenets of the protocol: (a) Vortex ring impulse is directly proportional to the characteristic length scale D, and (b) quasi-steady jet impulse from a constant mass flux source is inversely proportional to D.

Therefore, during jet initiation when vortex formation occurs, D should be increased in time to take advantage of the first tenet. If fluid transport is terminated at dimensionless time $\hat{T} \approx 4$, the sections above show that efficiency is maximized. However, to maximize impulse, one needs to extend the fluid transport beyond $\hat{T} \approx 4$ while decreasing the length scale D, per the second tenet. Here, the trailing jet behind the pinched-off vortex ring provides the primary source for fluid impulse.

Dabiri & Gharib (2005b) analyzed measurements of transmitral blood flow for patients with normal and pathological cardiac function, and they observed that the proposed protocol in Figure 3a was manifested in each case studied. In particular, the most severe pathology (i.e., reduced

E/A) exhibited a temporal trend of D versus $\hat{T}$ that indicated a dramatic transition from efficient propulsion to a propulsion that maximizes impulse (Figure 3b). A possible explanation for this transition is that for cases of severe dysfunction, efficient blood transport must be sacrificed to merely achieve the transport of mass and momentum required for the continued operation of the left heart. Kilner et al. (2000) and Kheradvar et al. (2007) have confirmed that a baseline fluid impulse is required in the transmitral blood flow to facilitate proper recoil and filling of the left ventricle.

4. Discussion

This work has shown that functionally and morphologically diverse biological fluid transport systems can be designed and tuned using simple rules in accordance with the dynamics of vortex ring formation. The framework introduced here for studying biological jet flows has the potential to connect functionally disparate systems in comparative studies, and to improve our understanding of jet flow pathologies relative to normal function. We also suggest that the analytical method developed here can be used to develop a connection between the observed robustness of jet-based vortex ring formation in aquatic propulsion (Linden & Turner 2004) and the previously discovered optimal Strouhal parameter for flapping-based vortex ring formation in insect and bird flight (Taylor et al. 2003). In an analysis of the latter group, the jet exit diameter of relevance in the present paper will be replaced with considerations for the time-dependent flap morphology and flapping kinematics.

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